분리적 그래프 분할을 이용한 다중 능동 윤곽선

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Multiple Active Contours Using Divisive Graph Cuts

Jong-Sung Kim*, Ki-Sang Hong**
Dept. Electronic and Electrical Engineering, POSTECH
*kimjs@postech.ac.kr, ** hongks@postech.ac.kr

Abstract: This paper presents a new algorithm for multiple active contours based on graph cuts to detect the optimal boundary of multiple regions in a given image. The problem is stated as a partitioning problem and solved using divisive graph cuts, which overcome the computational complexity of graph cuts for multiple active contours. The proposed algorithm can find multiple contours without multiple initial contours and seeds. In addition, the number of regions can be estimated as a part of the algorithm when it is not given a priori. Experimental results for synthetic and real images are presented to demonstrate the effectiveness of the proposed algorithm.

Keywords: multiple active contours, graph cuts, divisive partitioning.

Ⅰ. Introduction

Detecting boundaries and regions in images is an important issue in computer vision. Boundaries and regions are useful visual cues for several vision applications, such as object recognition, surveillance, video compression, and so on.

Active contours detect the boundary of single or multiple regions by evolving contours toward boundaries represented by edges as in snakes [8] and geodesic active contours [4] or toward the boundary of homogenous regions as in the Chan-Vese model [5, 15]. The edge and region-based active contour models can be unified in order to improve the performance of active contours as in geodesic active regions [10].

Level set methods [12] have been widely used to implement active contours algorithms, since they can implicitly represent a contour with the zero level of a signed distance function. However, level set methods have some undesirable properties. For example, they can find only a local minimum near given initial solution with slow convergence speed. Besides, it is impossible to keep the evolving level set function close to a signed distance function without re-initialization. Graph cuts [1, 3, 9] can be an alternative for level set methods as in geo-cuts [2, 8] and graph cuts based on active contours [16], but they also have limitations, particularly for handling multiple contours, since finding multiple cuts is NP-hard even for multi-way graph cuts [1]. In addition, it is difficult to initialize multiple contours for graph cuts-based active contours without prior knowledge, such as user-selected seeds or the number of regions.

This paper presents a new algorithm for multiple active contours based on graph cuts to detect the optimal boundary of multiple regions without multiple initial contours and seeds. The problem is stated as a partitioning problem and a new concept of divisive graph cuts is introduced to solve the problem. The proposed algorithm deforms a contour to partition a given image into two regions, which is conducted by minimizing an energy function for edge and region-based active contours via binary graph cuts. Then, this procedure is repeatedly applied to current regions until the number of regions reaches to a predefined value or some stopping criteria are met. In this way, the proposed algorithm can find optimal multiple contours without multiple initial contours and seeds. It does not suffer from the computational complexities for initializing and finding multiple contours. The number of regions can be estimated as a part of the algorithm when it is not given a priori.

The remainder of the paper is organized as follows: Section Ⅱ introduces active contours using graph cuts for two regions. Section Ⅲ describes the concept of divisive graph cuts and how to apply it to extend the graph cuts-based active contours for multiple regions. Sections Ⅳ and Ⅴ include the experimental results and the conclusions.

Ⅱ. Active Contours Using Graph Cuts

Assume that a given image I can be partitioned into two non-overlapping regions R_A and R_B, subject to $R_A \cap R_B = \varnothing$ and $R_A \cup R_B = \Omega$, where Ω is the domain of image, and the image model is the piecewise constant Mumford-Shah model [9] as in [15]. Model parameters are denoted by $\theta = (\theta_A, \theta_A)$.

The idea of active contours is to evolve a contour C close to the boundary of two regions. In this work, the continuous energy function for active contours is defined by

$$E(C,\theta) = -\lambda \iint_{\mathbf{x}\in\{R_A,R_B\}} \log p\left(I(\mathbf{x})|\theta\right) dxdy + \int_0^1 g\left(\left|\nabla I\left(C(s)\right)\right|\right)\left|C'(s)\right| ds, \quad (1)$$

where λ is a positive constant, $\mathbf{x}$ denotes the pixel location, p a probability density function, and g an edge-stopping function [4]. In this work, p is set to a Gaussian distribution function, the edge-stopping function is defined by $g(z) = \exp\left(-\gamma z^2\right)$ with $\gamma = 0.25$. The first term is the internal force minimizing the fidelity error of region models and the second term the external force minimizing the length of contour [4]. If a good initial contour close to the true boundary and good model parameters are given a priori, a locally optimal contour can be obtained by evolving the initial contour via level set methods. However, level set methods have some undesirable properties as described in the preceding section. In the proposed algorithm, the evolution of contours via level sets is substituted to the search of optimal contours via graph cuts as in [2]. For this end, the energy function of (1) is redefined on a graph G by using the Potts model [14]

$$E(f,\theta) = -\lambda \sum_{\mathbf{x}\in\{R_A,R_B\}} \log p\left(I(\mathbf{x})|\theta\right) + \sum_{\{\mathbf{x},\mathbf{y}\}\in N} g_{\{\mathbf{x},\mathbf{y}\}} \cdot w_{\{\mathbf{x},\mathbf{y}\}} \cdot \delta\left(f_{\mathbf{x}}, f_{\mathbf{y}}\right), \quad (2)$$

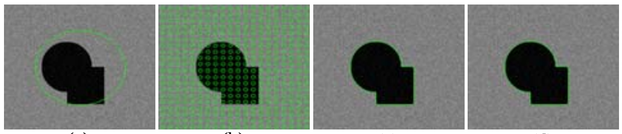
where $f = \left(f_1, \ldots, f_{\mathbf{x}}, \ldots, f_{|\Omega|}\right)$ is a binary label vector of which element $f_{\mathbf{x}} \in \{0, 1\}$ assigns the pixel $\mathbf{x}$ to one of two regions, N the set of neighboring pixels, $g_{\{\mathbf{x},\mathbf{y}\}}$ the edge-weights corresponding to the edge stopping function, defined by

$$g_{\{\mathbf{x},\mathbf{y}\}} = \exp\left(-\gamma\left|I_{\mathbf{x}} - I_{\mathbf{y}}\right|^2\right), \quad (3)$$

$w_{\{\mathbf{x},\mathbf{y}\}}$ the edge-weights approximating the length of contour, of which details are in [2], and

$$\delta\left(f_{\mathbf{x}}, f_{\mathbf{y}}\right) = \begin{cases} 1 & \text{if } f_{\mathbf{x}} \neq f_{\mathbf{y}} \\ 0 & \text{otherwise} \end{cases}. \quad (4)$$

The energy function of (2) is minimized with respect to f and θ alternatively using binary graph cuts and a parameter estimation technique. Figure 1 illustrates this procedure with an example of active contours for one contour and two regions of different gray values. The figure shows that the final contours do not depend on the location of initial contour.

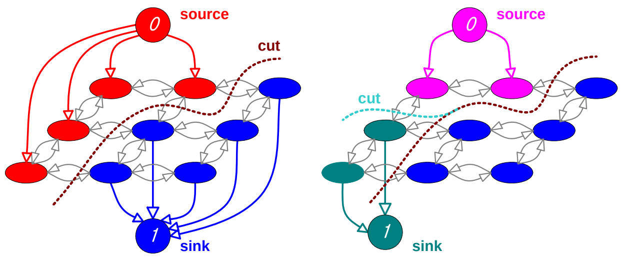


(a) (b) (c) (d)

Figure 1. An example of active contours for one contour and two regions with the proposed algorithm (λ =1.0). (a) and (b) show different initial contours. (c) and (d) show the final contours corresponding to (a) and (b), respectively.

Ⅲ. Multiple Active Contours Using Divisive Graph Cuts

Multiple active contours may be stated as a multi-label problem and solved using multi-way graph cuts [6]. However, multi-label problems are NP-hard even though multi-way graph cuts are applied [1]. The computational complexity for multi-way cuts is very high compared to binary graph cuts. In addition, it is strictly prerequisite to set multiple initial contours close to the true boundary of multiple regions, or to select seed points as some hard constraints in order to assign each pixel to one of multiple regions without overlapping and vacuum. Evidently, these strict requirements can limit the applicability of active contour algorithms in some applications.



(a) A cut on G (b) A new cut on G

Figure 2. Divisive graph cuts. Binary graph cuts are repeatedly applied to some part of a graph to obtain a new cut.

To overcome the limitations of graph cuts for multiple active contours, a new concept of divisive graph cuts is introduced by combining divisive partitioning [7] and graph cuts. As illustrated in Figure 2, the idea of divisive graph cuts is to repeatedly apply binary graph cuts to some part of a graph in order to obtain a new cut partitioning that part without graph reconstruction. In this way, divisive graph cuts can obtain multiple cuts without the increase in the computational complexity. It is a general approach for obtaining multiple cuts even though it is applied to multiple active contours in this work. The proposed algorithm starts to find a contour partitioning a given image into two regions by finding a contour minimizing the energy function of (2). This procedure is recursively applied to current regions until the number of regions reaches a predefined value, denoted by M, or some stopping criteria are met. It can be summarized as the following four steps:

1. Select a region, whose cost is larger than other regions.
2. Find a contour to partition that region to two regions.
3. Accept the contour and the regions if some criteria are satisfied.
4. Stop the algorithm if the number of regions reaches to M or the regions are not accepted. Otherwise, go to step 1.

In step 1, the cost for some region R_i is with the energy of that region, defined by

$$e(R_i) = -\lambda \sum_{\mathbf{x}\in\{R_i\}} \log p\left(I(\mathbf{x})|\theta_i\right) + \sum_{\{\mathbf{x},\mathbf{y}\}\in N} g_{\{\mathbf{x},\mathbf{y}\}} \cdot w_{\{\mathbf{x},\mathbf{y}\}} \cdot \delta\left(f_{\mathbf{x}}, f_{\mathbf{y}}\right). \quad (5)$$

Two criteria are used in step 3. One is the distance between the means, denoted by μ, of the density functions of two regions, defined by

$$d\left(R_i, R_j\right) = \left\|\mu_i - \mu_j\right\|, \quad (6)$$

which measures the similarity between two regions. If the distance is very small, the current region should not be partitioned any more. The other is a measure of the skew of region, defined by

$$r(R_i) = \begin{cases} 1-|L_i|/|R_i| & \text{if } |R_i| \geq \varepsilon \\ 0 & \text{otherwise} \end{cases}, \quad (7)$$

where $|R_i|$ and $|L_i|$ are the area and the length of boundary of some region R_i, respectively, and ε a threshold for the area. This value will be close to zero as regions are skewed as thin strips. If one of the two regions is skewed, the current region should not be partitioned. Figure 3 illustrates this procedure with an example of multiple active contours obtained by the proposed algorithm for two contours and three regions with different gray values, where the number of regions is set to three.

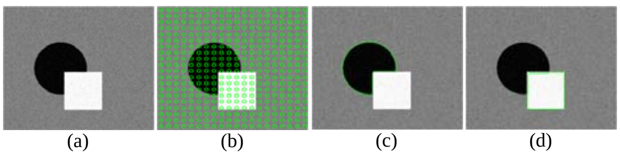


Figure 3. An example of multiple active contours for two contours and three regions with the proposed algorithm (λ =1.0, M=3). (a) shows the test image. (b) shows the initial contour. (c) and (d) show two final contours.

Ⅳ. Experimental Results

Numerical experiments were conducted with synthetic and real images to demonstrate the effectiveness of the proposed algorithm. All computations were carried out on a desktop PC with a 2.4GHz Intel P4 CPU executing C/C++ code. In all experiments, 5.0, and 0.5 were used for the threshold of two acceptance measures, d and r, respectively. In all experiments, the number of regions was set with M=7, in order to show that the proposed algorithm can estimate the number of regions. The proposed algorithm is implemented by using the graph cuts code of [1, 3, 7]. Multiple initial contours or seeds were not used for multiple active contours. For demonstration, the boundary of each region was drawn with different color at each image.

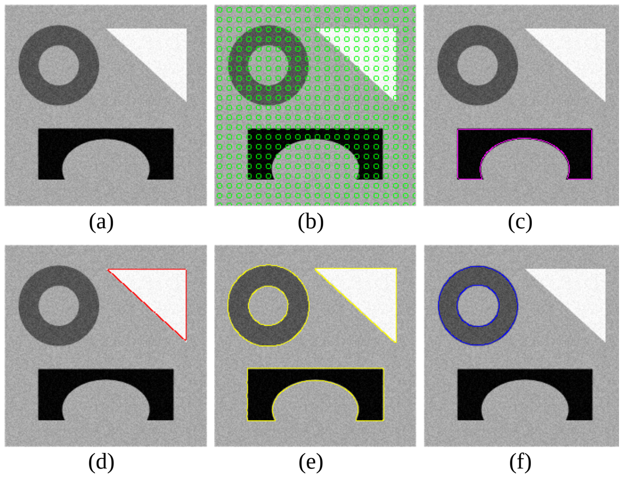


Figure 4. A result of multiple active contours with the proposed algorithm (λ =1.0, runtime=4.02s). (a) shows the synthetic image of size 227 x 227 with four regions. (b) shows the initial contour. (c), (d), (e), and (f) show the boundary of the regions whose labels are 0, 1, 2, and 3 in order.

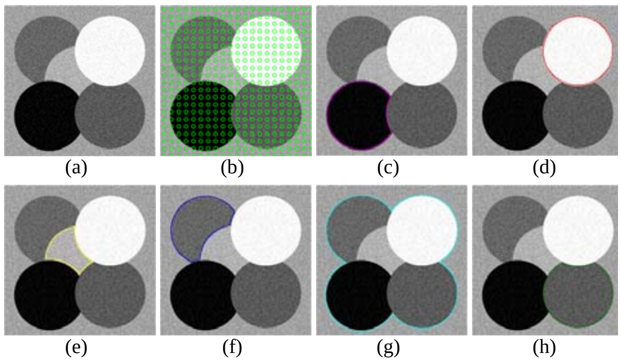


Figure 5. A result of multiple active contours with the proposed algorithm (λ =0.05, runtime=4.81s). (a) shows the synthetic image of size 227 x 227 with six regions. (b) shows the initial contour. (c), (d), (e), (f), (g) and (h) show the boundary of the regions whose labels are 0, 1, 2, 3, 4, and 5 in order.

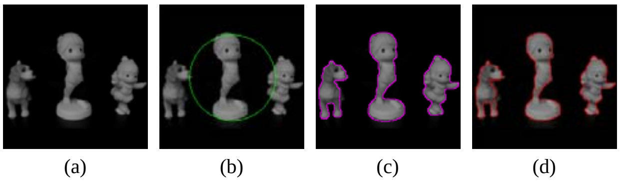


Figure 6. A result of multiple active contours with the proposed algorithm (λ =0.1, runtime=1.38s). (a) shows the real image of size 200 x 200 with two regions. (b) shows the initial contour. (c) and (d) show the boundary of two regions whose labels are 0 and 1 in order.

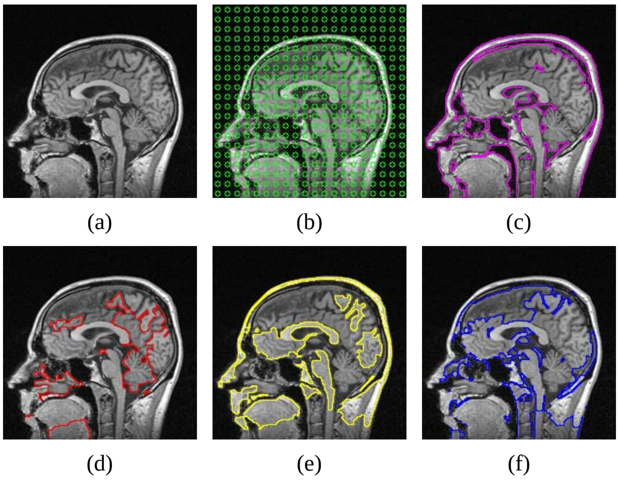


Figure 7. A result of multiple active contours with the proposed algorithm (λ =0.01, runtime=3.33s). (a) shows the real image of size 200 x 200 with multiple regions. (b) shows the initial contour. (c), (d), (e), and (f) show the boundary of four regions whose labels are 0, 1, 2, and 3 in order.

V. Conclusions

This paper presents a new algorithm for multiple active contours using graph cuts. The limitations of graph cuts for multiple active contours are solved using divisive graph cuts. The proposed algorithm successfully obtains the boundary of multiple regions without multiple initial contours and user-selected seeds. The number of regions can be estimated as a part of the algorithm.

The further work is to extend the proposed algorithm to use other visual cues, such as color, texture, and motion, for active contours.

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